

Efficient single-photon-assisted entanglement concentration for partially entangled photon pairs

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We present two realistic entanglement concentration protocols (ECPs) for pure partially entangled photons. A partially entangled photon pair can be concentrated to a maximally entangled pair with only an ancillary single photon in a certain probability, while the conventional ones require two copies of partially entangled pairs at least. Our first protocol is implemented with linear optics and the second one is implemented with cross-Kerr nonlinearities. Compared with other ECPs, they do not need to know the accurate coefficients of the initial state. With linear optics, it is feasible with current experiment. With cross-Kerr nonlinearities, it does not require the sophisticated single-photon detectors and can be repeated to get a higher success probability. Moreover, the second protocol can get the higher entanglement transformation efficiency and it maybe the most economical one by far. Meanwhile, both of protocols are more suitable for multi-photon system concentration, because they need less operations and classical communications. All these advantages make two protocols be useful in current long-distance quantum communications.

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I. INTRODUCTION

Entanglement plays an important role in current quantum information processing, such as quantum computation [1], quantum key distribution [2, 3], quantum teleportation [4], controlled teleportation [5], dense coding [6], and quantum-state sharing [7]. In the past ten years, a large number of experiments have been reported that quantum computation and quantum communication are more powerful in many aspects than their classical counterparts. In order to complete such quantum information processing protocols, the maximally entangled states are usually required. However, in a practical transmission and storage, the entanglement inevitably will contact with the environment, and the noise will make the entanglement degrade. Generally speaking, the maximally entangled state such as Bell state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$ may become a mixed state. That is

$$\rho = F|\phi^+\rangle\langle\phi^+| + (1-F)|\psi^+\rangle\langle\psi^+|. \quad (1)$$

Here $|\psi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle)$. It also may become a partially entangled state $|\varphi\rangle = \alpha|H\rangle|H\rangle + \beta|V\rangle|V\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$. Here $|H\rangle$ ($|V\rangle$) represents the horizontal (vertical) photon polarization.

Entanglement purification can distill a set of mixed entangled states into a subset of highly entangled states with local

operation and classical communication [8–15]. However, it can only improve the quality of the mixed state and can not get the maximally entangled state. On the other hand, entanglement concentration, which will be detailed can be used to convert the partially entangled pairs to the maximally entangled ones [16–27]. In 1996, Bennett *et al.* proposed the first entanglement concentration protocol (ECP), named Schmidt decomposition protocol. In their protocol, they use the collective measurements which are difficult to manipulate in experiment [16]. Bose *et al.* also proposed an ECP based on entanglement swapping [17]. But their protocol requires the collective Bell-state measurement. Moreover, they need to know the coefficients in order to reconstruct the same entangled states. In 2001, Zhao *et al.* and Yamamoto *et al.* proposed two similar ECPs independently with polarization beam splitters (PBSs) [19–22]. They have simplified the schmidt projection method and adopted the parity check to substitute the original collective measurement. Here we call it PBS1 protocol. This idea has been developed to reconstruct the ECP with cross-Kerr nonlinearity, which can be used to construct the quantum nondemolition detectors (QND) [25]. Here we call it QND1 protocol.

The current ECPs have in common that, for instance, in Refs. [19, 21, 23–27] they need at least two copies of less-entangled (or say partially entangled) pairs initially. But after performing the protocols, at most one pair of maximally entangled state can be obtained, or both of each should be discarded, according to the measurement results by classical communication. Local operation and classical communica-

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tion can not be used to increase entanglement. Therefore, these ECPs are essentially the transformation of entanglement and the previous works of entanglement concentration are not optimal. Actually, two copies of less-entangled pairs are not necessary. Using one copy of less-entangled pair to distillate high quality entanglement has been proposed in continuous variables system. In Ref. [28], Opatrny *et al.* showed the improvement on teleportation of continuous variables by photon subtraction via conditional measurement. In 2008, He and Bergou proposed a general probabilistic approach for transforming a single copy of discrete entanglement state without classical communication [29].

In this paper, we describe two single-photon-assisted ECPs in which only one pair of less-entangled state and one single photon are required. The two ECPs are focused on the practical discrete less-entangled photon pairs and they are implemented with linear optics and cross-Kerr nonlinearity, respectively. Comparing with current ECPs, the single-photon-assisted ECPs are more economical. In the first ECP, with linear optics, it can reach the same success probability as the protocol of Ref. [21], but requires only one pair of less-entangled state. In the second ECP, with the help of cross-Kerr nonlinearity, it can be repeated to get a higher success probability. These advantages make the two protocols more feasible in practical applications.

This paper is organized as follows: in Sec.II, we first explain the basic principle of single-photon-assisted ECP with linear optics. We call it PBS2 protocol. In Sec.III, we extend this protocol to the system of cross-Kerr nonlinearity. We call it QND2 protocol. We show that sophisticated single-photon detectors are not required and the discarded items in PBS2 protocol can also be reused to perform concentration. The higher success probability can be obtained than other protocols. In Sec. IV, we first calculate the entanglement transformation efficiency, and then make a discussion and summary.

II. SINGLE-PHOTON-ASSISTED ENTANGLEMENT CONCENTRATION WITH LINEAR OPTICS

The basic principle of our PBS2 protocol is shown in Fig.1. The less-entangled pair of photons emitted from S_1 are sent to Alice and Bob. The photon a belongs to Alice and b belongs to Bob. The initial photon pair is in the following unknown state:

$$|\Phi\rangle_{ab1} = \alpha|H\rangle_{a1}|H\rangle_{b1} + \beta|V\rangle_{a1}|V\rangle_{b1}. \quad (2)$$

The another source S_2 emits a single photon with the form of

$$|\Phi\rangle_{a2} = \alpha|H\rangle_{a2} + \beta|V\rangle_{a2}. \quad (3)$$

Here $|\alpha|^2 + |\beta|^2 = 1$. $a1$, $b1$, and $a2$ are different spatial modes.

The initial state of the three photons can be written as:

$$\begin{aligned} |\Psi\rangle &= |\Phi\rangle_{ab1} \otimes |\Phi\rangle_{a2} \\ &= \alpha^2|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2} + \alpha\beta|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2} \\ &\quad + \alpha\beta|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2} + \beta^2|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}. \end{aligned} \quad (4)$$

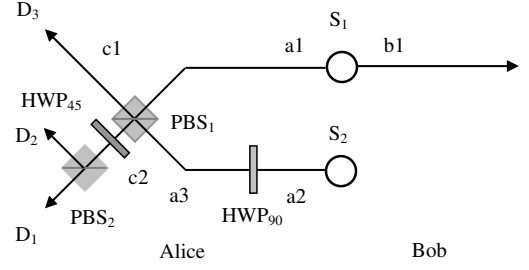


FIG. 1: A schematic drawing of the single-photon-assisted ECPs with linear optics. S_1 is the partial entanglement source and S_2 is the single photon source. PBSs transmit the horizontal polarization component and reflect the vertical component. HWP_{90} and HWP_{45} can rotate the polarization of the state by 90° and 45° , respectively.

Alice first rotates the polarization state of the single photon $|\Phi\rangle_{a2}$ by 90° by half-wave plate (HWP_{90} in Fig.1). Then the state can be rewritten as:

$$\begin{aligned} |\Psi'\rangle &= |\Phi\rangle_{ab1} \otimes |\Phi'\rangle_{a3} \\ &= \alpha^2|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a3} + \alpha\beta|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a3} \\ &\quad + \alpha\beta|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a3} + \beta^2|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a3} \\ &= \alpha^2|H\rangle_{a1}|V\rangle_{a3}|H\rangle_{b1} + \beta^2|V\rangle_{a1}|H\rangle_{a3}|V\rangle_{b1} \\ &\quad + \alpha\beta(|H\rangle_{a1}|H\rangle_{a3}|H\rangle_{b1} + |V\rangle_{a1}|V\rangle_{a3}|V\rangle_{b1}). \end{aligned} \quad (5)$$

From above equation, it is evident that the items $|H\rangle_{a1}|H\rangle_{a3}|H\rangle_{b1}$ and $|V\rangle_{a1}|V\rangle_{a3}|V\rangle_{b1}$ will lead the two output modes $c1$ and $c2$ both exactly contain only one photon. However, item $|H\rangle_{a1}|V\rangle_{a3}|H\rangle_{b1}$ will lead two photons both in $c2$ mode, and item $|V\rangle_{a1}|H\rangle_{a3}|V\rangle_{b1}$ will lead both photons in $c1$ mode. Therefore, by choosing the three-mode cases, i.e. each modes of $c1$, $c2$ and $b1$ exactly contain and only contain one photon, the initial state is projected into a maximally three-photon entangled state:

$$|\Psi''\rangle = \frac{1}{\sqrt{2}}(|H\rangle_{c1}|H\rangle_{c2}|H\rangle_{b1} + |V\rangle_{c1}|V\rangle_{c2}|V\rangle_{b1}), \quad (6)$$

with a probability of $2|\alpha\beta|^2$.

In order to generate a maximally entangled Bell-state between Alice and Bob, they could perform a 45° polarization measurement onto the photon $c2$. In Fig.1, with quarter-wave plate (HWP_{45} in Fig.1), it can make:

$$\begin{aligned} |H\rangle_{c2} &\rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{c2} + |V\rangle_{c2}), \\ |V\rangle_{c2} &\rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{c2} - |V\rangle_{c2}). \end{aligned} \quad (7)$$

After the rotation, Eq. (6) will evolve to

$$\begin{aligned} |\Psi'''\rangle &= \frac{1}{2}(|H\rangle_{c1}|H\rangle_{b1} + |V\rangle_{c1}|V\rangle_{b1})|H\rangle_{c2} \\ &\quad + (|H\rangle_{c1}|H\rangle_{b1} - |V\rangle_{c1}|V\rangle_{b1})|V\rangle_{c2}. \end{aligned} \quad (8)$$

Now Alice lets the photon $c2$ pass through the PBS₂. Clearly, if the detector D_1 fires, the photon pair is left in the state as

$$|\phi^+\rangle_{a1b1} = \frac{1}{\sqrt{2}}(|H\rangle_{c1}|H\rangle_{b1} + |V\rangle_{c1}|V\rangle_{b1}). \quad (9)$$

If the detector D_2 fires, the photon pair is left in the state as

$$|\phi^-\rangle_{a1b1} = \frac{1}{\sqrt{2}}(|H\rangle_{c1}|H\rangle_{b1} - |V\rangle_{c1}|V\rangle_{b1}). \quad (10)$$

Both of Eqs. (9) and (10) are the maximally entangled states. One of them say Alice or Bob only needs to perform a phase flip to convert Eq. (10) to (9), and the whole concentration process is finished. It has the success probability of $2|\alpha\beta|^2$, which is the same as Ref. [21]. During the whole process, they do not require two copies of less-entangled pair, and only one pair and a single photon are required. Meanwhile, they do not need to know the exact coefficients of the initial states $|\Phi\rangle_{a1b1}$ and $|\Phi\rangle_{a2}$, but only require them to equal.

From Fig.1, it is straightforward to extend this protocol to reconstruct maximally entangled multi-partite Greenberg-Horne-Zeilinger (GHZ) states from partially entangled GHZ states.

The partially multi-partite entangled GHZ state with N photons can be written as:

$$|\Phi\rangle_N = \alpha|H\rangle_A|H\rangle_B \cdots |H\rangle + \beta|V\rangle_A|V\rangle_B \cdots |V\rangle. \quad (11)$$

Here the subscriptions $A, B \cdots$ denote the parties Alice, Bob, Charlie etc.. Each one owns one photon. With the same principle described above, Alice needs another single photon with the same form of Eq. (3). After passing through the PBS, if the output modes of Alice's PBS both contain exactly one photon, then the whole system collapses into a $N + 1$ maximally entangled state. Then with the same principle, they can obtain the N -photon maximally entangled state by measuring one photon after rotating 45° .

Interestingly, this protocol seems more feasibly for the concentration of multi-photon GHZ system. First, in this protocol, only Alice needs to perform this protocol, while in the conventional protocols [19, 21, 25], all the parties should perform the same operations with Alice. Second, in this protocol, only Alice asks other parties to retain or discard their photons, and they do not need to check their measurement results. It greatly simplifies the complication of classical communication. Third, multi-photon GHZ states are more difficult to generate in current condition. We can have the same success probability but only need one pair of less-entangled GHZ state.

III. SINGLE-PHOTON-ASSISTED ENTANGLEMENT CONCENTRATION WITH CROSS-KERR NONLINEARITY

So far, we have briefly described the single-photon-assisted ECP based on linear optics. In above description, Alice exploits the PBS and sophisticated single-photon detectors to distinguish $|HH\rangle$ and $|VV\rangle$ from $|HV\rangle$ and $|VH\rangle$. It is essentially the parity-check measurement of polarization photons.

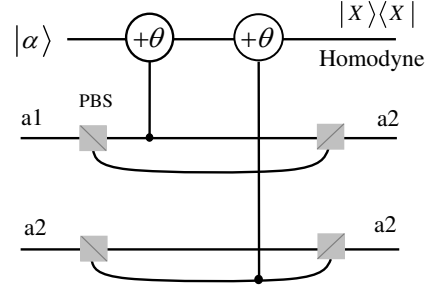


FIG. 2: The schematic drawing of the principle of the quantum non-demolition detector (QND) constructed by the cross-Kerr nonlinearity. It can also be shown in Ref. [25]. One can distinguish the state $|HH\rangle$ and $|VV\rangle$ from $|HV\rangle$ and $|VH\rangle$ by different phase shift of the coherent state.

Unfortunately, with current technology, sophisticated single-photon detectors are not likely to be available, that makes this protocol cannot be achieved simply with only linear optics. In this section, we will introduce the cross-Kerr nonlinearity to construct QND, which can also be used to implement this protocol. Cross-kerr nonlinearity has been widely studied in the construction of CNOT gate [30], performance of complete Bell-state analysis [31], entanglement purification [13] and so on [32–40]. The Hamiltonian of a cross-Kerr nonlinear medium can be written as $H = \hbar\chi\hat{n}_a\hat{n}_b$. Here the $\hbar\chi$ is the coupling strength of the nonlinearity, which is decided by the material of cross-Kerr. The $\hat{n}_a(\hat{n}_b)$ is the number operator for mode $a(b)$.

If we consider a coherent state $|\alpha\rangle$ and a single photon $|\psi\rangle = \gamma|0\rangle + \delta|1\rangle$ interact with the cross-Kerr nonlinearity, the whole system can be described as [30, 31]:

$$\begin{aligned} U_{ck}|\psi\rangle|\alpha\rangle &= (\gamma|0\rangle + \delta|1\rangle)|\alpha\rangle \\ &= \gamma|0\rangle|\alpha\rangle + \delta|1\rangle|\alpha e^{i\theta}\rangle. \end{aligned} \quad (12)$$

Here $|0\rangle$ and $|1\rangle$ are the Fock states, which mean no photon and one photon respectively. $\theta = \chi t$ and t is the interaction time. It is obvious that the phase shift of coherent state is directly proportional to the number of photons.

In Fig.3, we adopt the QND to substitute the PBS. Then the whole system $|\Phi\rangle_{a1b1} \otimes |\Phi'\rangle_{a3}$ with the coherent state $|\alpha\rangle$ can be written as:

$$\begin{aligned} |\Psi'\rangle|\alpha\rangle &= |\Phi\rangle_{a1b1} \otimes |\Phi'\rangle_{a3}|\alpha\rangle \\ &= (\alpha^2|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a3} + \alpha\beta|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a3} \\ &\quad + \alpha\beta|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a3} + \beta^2|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a3})|\alpha\rangle \\ &\rightarrow \alpha^2|H\rangle_{a1}|V\rangle_{a3}|H\rangle_{b1}|\alpha e^{i2\theta}\rangle + \beta^2|V\rangle_{a1}|H\rangle_{a3}|V\rangle_{b1}|\alpha\rangle \\ &\quad + \alpha\beta(|H\rangle_{a1}|H\rangle_{a3}|H\rangle_{b1} + |V\rangle_{a1}|V\rangle_{a3}|V\rangle_{b1})|\alpha e^{i\theta}\rangle. \end{aligned} \quad (13)$$

In above evolution, the items $|H\rangle_{a1}|V\rangle_{a3}$ and $|V\rangle_{a1}|H\rangle_{a3}$ pick up the phase shift with 2θ and no phase shift, respectively. But the items $|H\rangle_{a1}|H\rangle_{a3}$ and $|V\rangle_{a1}|V\rangle_{a3}$ both pick up the phase shift with θ . Therefore, if the phase shift of homodyne measurement is θ , Alice asks Bob to keep the whole state. Otherwise

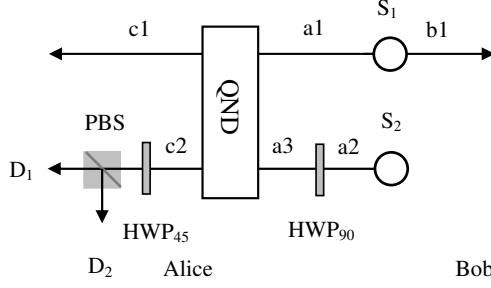


FIG. 3: The schematic drawing of the ECP with QND. Here we substitute the PBS shown in Fig.1 with QND. This ECP can obtain a higher success probability.

they discard the state. The remained state essentially is the state described in Eq. (6). Therefor, following the same step described above, one can ultimately obtain the maximally entangled state $|\phi^+\rangle_{c1b1}$ if D_1 fires, and get $|\phi^-\rangle_{c1b1}$ if D_2 fires.

In above description, Alice only picks up the instance in which the phase shift θ on his coherent state, and discards the other instances. If a suitable cross-Kerr medium is available and Alice can control the interaction time t exactly, which makes the phase shift $\theta = \pi$, in this way, one can not distinguish the phase shift 0 and 2π . The discarded items in the above equation are

$$|\Phi'\rangle = \alpha^2|H\rangle_{c1}|V\rangle_{a3}|H\rangle_{b1} + \beta^2|V\rangle_{c1}|H\rangle_{a3}|V\rangle_{b1}, \quad (14)$$

with the probability of $|\alpha|^4 + |\beta|^4$. Alice uses the HWP₄₅ to rotate the photon in $c2$ and finally it is detected by D_1 or D_2 . Eq. (14) becomes

$$|\Phi''\rangle = \alpha^2|H\rangle_{c1}|H\rangle_{b1} + \beta^2|V\rangle_{c1}|V\rangle_{b1}, \quad (15)$$

if D_1 fires, and becomes

$$|\Phi'''\rangle = \alpha^2|H\rangle_{c1}|H\rangle_{b1} - \beta^2|V\rangle_{c1}|V\rangle_{b1}, \quad (16)$$

if D_2 fires.

$|\Phi''\rangle$ and $|\Phi'''\rangle$ are both the partially entangled states which can also be used to reconcentrate to a maximally entangled states. For instance, if they get $|\Phi''\rangle$, Alice only needs to choose another single photon with the form of $\alpha^2|H\rangle_{a2} + \beta^2|V\rangle_{a2}$, and follows the same method described above. That is:

$$\begin{aligned} & (\alpha^2|H\rangle_{c1}|H\rangle_{b1} + \beta^2|V\rangle_{c1}|V\rangle_{b1}) \\ & \otimes (\alpha^2|H\rangle_{a2} + \beta^2|V\rangle_{a2}) \otimes |\alpha\rangle \\ & \rightarrow (\alpha^2|H\rangle_{c1}|H\rangle_{b1} + \beta^2|V\rangle_{c1}|V\rangle_{b1}) \\ & \otimes (\alpha^2|V\rangle_{a2} + \beta^2|H\rangle_{a2}) \otimes |\alpha\rangle \\ & \rightarrow \alpha^4|H\rangle_{a1}|V\rangle_{a3}|H\rangle_{b1}|\alpha e^{i2\theta}\rangle + \beta^4|V\rangle_{a1}|H\rangle_{a3}|V\rangle_{b1}|\alpha\rangle \\ & + (\alpha\beta)^2(|H\rangle_{a1}|H\rangle_{a3}|H\rangle_{b1} + |V\rangle_{a1}|V\rangle_{a3}|V\rangle_{b1})|\alpha e^{i\theta}\rangle. \end{aligned} \quad (17)$$

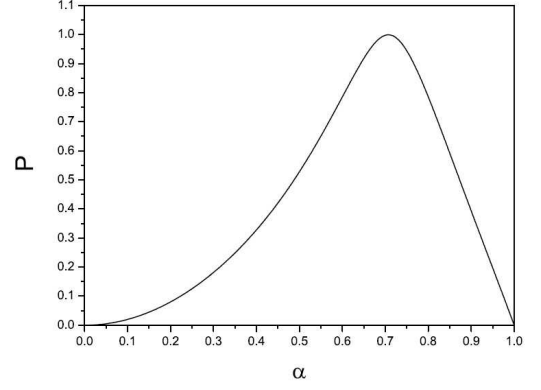


FIG. 4: The success probability P of getting a maximally entangled state after performing the QND2 protocol N times ($N \rightarrow \infty$) is altered with the entanglement of the initial partially entangled state, i.e. α . For numerical simulation, we let $N = 10$ as a good approximation.

Alice can also pick up the phase shift θ and get the same state as Eq. (6). The success probability of obtaining Eq. (6) is

$$P_2 = \frac{2|\alpha\beta|^4}{|\alpha|^4 + |\beta|^4}. \quad (18)$$

We can also get

$$\begin{aligned} P_3 &= \frac{2|\alpha\beta|^8}{|\alpha|^8 + |\beta|^8}, \\ &\dots \\ P_N &= \frac{2|\alpha\beta|^{2^N}}{|\alpha|^{2^N} + |\beta|^{2^N}}, \end{aligned} \quad (19)$$

where N is the iteration number of our concentration processes.

The total success probability to get a maximally entangled state from the initial partially entangled state is

$$P = P_1 + P_2 + \dots + P_N = \sum_{N=1}^{\infty} P_N. \quad (20)$$

Interestingly, if $\alpha = \beta = \frac{1}{\sqrt{2}}$, $P = \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^N} = 1$. But if $\alpha \neq \beta$, $P < 1$. Fig. 4 shows that the relationship between the coefficient of initial partially entangled state α and total success probability P . From Fig. 4, it is shown that the success probability is not a fixed value. It is related with the entanglement of the initial state, and it increases with the entanglement of the initial partially entangled state.

We should point out that during a practical operation, the longer interaction time will induce decoherence from losses. It will make the output state become a mixed one. Therefor, by controlling the longer interaction time to make the phase shift $\theta = \pi$ may not seem like an efficient way. Fortunately, another better alternative way is to rotate the coherent state in Eq. (13) by θ . After rotation, Eq. (13) becomes

$$\begin{aligned} & \rightarrow \alpha^2|H\rangle_{a1}|V\rangle_{a3}|H\rangle_{b1}|\alpha e^{i\theta}\rangle \\ & + \beta^2|V\rangle_{a1}|H\rangle_{a3}|V\rangle_{b1}|\alpha e^{-i\theta}\rangle \\ & + \alpha\beta(|H\rangle_{a1}|H\rangle_{a3}|H\rangle_{b1} + |V\rangle_{a1}|V\rangle_{a3}|V\rangle_{b1})|\alpha\rangle. \end{aligned} \quad (21)$$

From above description, if the coherent state picks up no phase shift, the remained state is also the same as Eq. (6). Otherwise, one can use the $|X\rangle\langle X|$ homodyne detection [30], which makes the $|\alpha e^{\pm i\theta}\rangle$ can not be distinguished. In this way, the discarded state is also the same as it is described in Eq. (14). Moreover, with the help of QND, this protocol can also be extended to multi-photon system, and be used to reconstruct maximally entangled multi-photon GHZ state. It has the same success probability as shown in Fig.4.

IV. DISCUSSION AND SUMMARY

By far, we have fully described our protocols with both PBS and QND. In each protocol, we only require one pair of less-entangled photons and a single photon. It is known that local operation and classical communication can not increase the entanglement. Therefore, entanglement concentration is essentially the transformation of entanglement. We define the entanglement transformation efficiency η as:

$$\eta = \frac{E_c}{E_0}. \quad (22)$$

Here E_0 is the entanglement of initial partially entangled state, and E_c is the entanglement of the state after performing concentration one time. E_c can be described as:

$$E_c = P_s \times 1 + (1 - P_s) \times E'. \quad (23)$$

The first item of Eq. (23) means that after concentration, we get the maximally entangled state with the success probability P_s . The second item means that the concentration is failure, and we get a more less-entangled pair. Obviously, if we use the PBS to perform the concentration, the second item is 0 for it collapses to a separated state $|HV\rangle$ or $|VH\rangle$ in different spatial modes [19, 21]. For two-body pure entangled state, Von Neumann entropy is suitable to describe the entanglement. Therefore, the entanglement of the initial state in Eq. (2) can be described as:

$$E = -|\alpha|^2 \log_2 |\alpha|^2 - |\beta|^2 \log_2 |\beta|^2. \quad (24)$$

We calculate the η of PBS1 protocol as [21]:

$$\eta_{PBS1} = \frac{2|\alpha\beta|^2 \times 1}{2E} = \frac{|\alpha\beta|}{E}. \quad (25)$$

The '2' in the denominator means that initially we need two copies of less-entangled states with entanglement E .

For QND1 protocol [25],

$$\eta_{QND1} = \frac{E'_{QND1}}{2E}, \quad (26)$$

with

$$\begin{aligned} E'_{QND1} &= 2|\alpha\beta|^2 \\ &+ (|\alpha|^4 + |\beta|^4) \left[-\frac{|\alpha|^4}{|\alpha|^4 + |\beta|^4} \log_2 \frac{|\alpha|^4}{|\alpha|^4 + |\beta|^4} \right. \\ &\quad \left. - \frac{|\beta|^4}{|\alpha|^4 + |\beta|^4} \log_2 \frac{|\beta|^4}{|\alpha|^4 + |\beta|^4} \right]. \end{aligned} \quad (27)$$

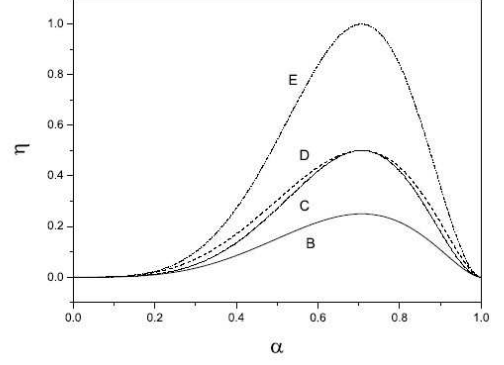


FIG. 5: The entanglement transformation efficiency η is altered with the coefficient α after performing each protocols one time. Curves B, C, D and E correspond to the protocols of PBS1 [21], QND1 [25], PBS2 and QND2 respectively. All the curves show that η increases with the entanglement of the initial entangled state. The QND2 protocol has the highest transformation efficiency. It can reach maximum value 1 when $\alpha = \frac{1}{\sqrt{2}}$.

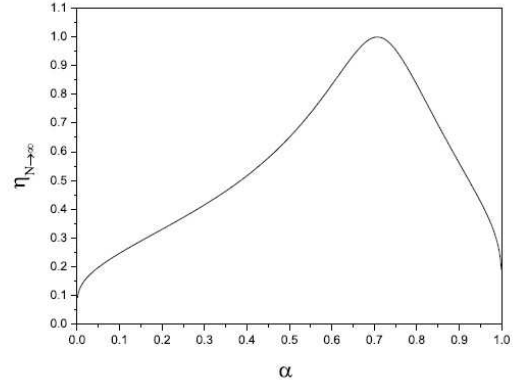


FIG. 6: The entanglement transformation efficiency η plotted against α after performing each protocols N times ($N \rightarrow \infty$) in QND2 protocol. For numerical simulation, we let $N = 10$ as a good approximation.

In our protocol, we only need one pair of less-entangled state to perform the protocol. Therefore, in PBS2 protocol,

$$\eta_{PBS2} = \frac{2|\alpha\beta|^2}{E} = 2\eta_{PBS1}, \quad (28)$$

and in QND2 protocol,

$$\eta_{QND2} = \frac{E'_{QND1}}{E} = 2\eta_{QND1}. \quad (29)$$

The relationship between the coefficient α and entanglement transformation efficiency is shown in Fig.5. It is shown that η is also not a fixed value, but increases with the initial entanglement. In QND2 protocol, η can reach the max value 1 with

$\alpha = \frac{1}{\sqrt{2}}$, which means that the initial one is the maximally entangled state. But in traditional protocols [21, 25], $\eta \leq 0.5$.

We also calculate the limit of entanglement transformation efficiency of QND2 protocol, by iterating the protocol N ($N \rightarrow \infty$) times.

$$\eta_{QND2}^{N \rightarrow \infty} = \frac{\sum_{N=1}^{\infty} E_N P_N}{E_0} = \frac{P}{E_0}. \quad (30)$$

The E_N means the entanglement of remained state after performing successful concentration in N th iteration. It is a maximally entangled state with $E_N = 1$. Fig.6 shows the relationship between α and transformation probability η . Obviously, η is monotone increasing with the entanglement of the initial state, and can get the max value 1 when the initial state is maximally entangled one, as $\alpha = \frac{1}{\sqrt{2}}$.

In this paper, the basic elements for us to complete the task are the PBS and QND. In fact, both of them act as the same role, that is parity check. In Refs [21] and [25], they also resort to PBS and QND to perform the concentration. But in each step, they require two pairs of less-entangled states. Our protocol shows that with only one pair of less-entangled state and a single photon, we can also achieve this task. This good feature makes these protocols have a higher entanglement transformation efficiency than others. In the processing of describing our concentration protocol, we exploit entanglement source and single photon source. An idea single photon source should exactly emit one and only one photon when the device is triggered. However, no single photon source and entanglement source will be ideal. In current technology, practical pulse generated by a source may contain no photons or multiple photons, with different probability. We denote P_m as the probability of emitting m photons. Interestingly, P_0 means no photon and will not give rise to errors. It only decreases the success probability of the protocol. Because only two photons can not satisfy the three-mode cases. However, $m \geq 2$ will give rise to errors. For example, $m = 2$ will lead both of the modes $c1$ and $c2$ contain one photon in Fig.1 which is a success event in our protocol. Fortunately, it is possible to make the probability of such events rather small. In Ref. [41], it was reported that a single photon source whose P_0 and P_2 are 14% and 0.08% respectively. Current spontaneous parametric down-conversion entanglement source is analogy with the single photon source. It generates entangled pair with the form of [19]

$$|\Upsilon\rangle = \sqrt{g}(|vac\rangle + \gamma|\phi^+\rangle + \gamma^2|\phi^+\rangle^2 + \dots). \quad (31)$$

The multi-photon items $|\phi^+\rangle^2$ can also cause errors. In practical teleportation experiments, the $\gamma^2 \sim 10^{-4}$ [42, 43], and the errors of multi-photon items is negligible.

In Sec.III, we exploit cross-Kerr nonlinearity to implement our protocol. Although a lot of works have been studied in the area of cross-Kerr nonlinearity [25, 27, 30–40], we should acknowledge that it is still a quite controversial assumption for a clean cross-Kerr nonlinearity in the optical single-photon

regime. In 2002, Kok *et al.* pointed out that the Kerr phase shift is only $\tau \approx 10^{-18}$ in the optical single-photon regime [44, 45]. In 2003, Hofmann showed that with a single two-level atom in a one-sided cavity, a large phase-shift of π can be achieved [46]. Gea-Banacloche showed that large shifts via the giant Kerr effect with single-photon wave packet is impossible in current technology [47]. The results of previous works of Shapiro and Razavi are consistent with Gea-Banacloche [48, 49]. Recently, He *et al.* discussed the feasibility of QND relies on the compatibility of small phase shift with large coherent state amplitude. They developed a general theory of the iteration between continuous-mode photonic pulses and applied it to the case of single photon interacting with a coherent state. They showed that if the pulses can fully pass through each other and the unwanted transverse-mode effects can be suppressed, the high fidelities, nonzero conditional phases, and high photon numbers are compatible [37]. Recent research also showed that using weak measurement, it is possible to amplify a cross-Kerr phase shift to an observable value, which is much larger than the intrinsic magnitude of the single-photon-level nonlinearity [50].

In summary, we present two different protocols for nonlocal entanglement concentration of partially entangled states. We exploit both PBS and cross-Kerr nonlinearity to achieve the task. Our protocols have several advantages: first, they do not need to know the exactly efficiency α and β of the less-entangled pairs. Second, they also do not resort to the collective measurement. Third, with QND, it does not require the parties to adopt the sophisticated single-photon detectors, and it can be iterated to get a higher success probability. Fourth, compared with the previous works, the most significant advantage is that in each step we only need one pair of less-entangled state. It provides our protocols to obtain more higher entanglement transformation efficiency than others. Fifth, these protocols are more feasible for multi-photon GHZ state concentration, because they greatly reduce the practical operations and simplify the complication of classical communication for each parties. All these advantages may make our protocols be more useful in practical applications.

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